Model Answers to Paper Code AS–2404

Master of Business Administration (First Semester) Examination, 2014

Paper : Second

QUANTITATIVE METHODS

Note: Attempt both the sections as directed.

Section – A

Note: Attempt all the questions. This section contains *Ten* short answer type questions having 2 Marks each. (10x2 = 20 Marks)

1. Different mathematicians, scientists have defined statistics in many ways. Some of the widely accepted definition is as follows; "Statistics is a body of methods making wise decisions in the face of uncertainity" By **Wallis & Roberts.** "Numerical statements of facts in any department of enquiry placed in the relation to each other".B**y Bowley.** "By statistics, we mean aggregate of facts affected to a marked extent by multiplicity of causes numerically expressed, enumerated or estimated according to a reasonable standard of accuracy, collected in a systematic manner for a pre-determined purpose and placed in relation to each other. **By Prof. Horace Secrist.**

<u>CHARACTERISTICS OF STATISTICS:-</u>

a) Statistics is a aggregate of Facts.
b) The Data is Numerically expressed.
c) The Statistical methods are affected by multiplicity of causes.
d) The data is, enumerated or estimated according to a reasonable standard of accuracy.
e) Data is collected in a systematic manner.
f) Data collected for pre-determined purpose.
g) The Data is arranged in relation to each other.

2. In mathematics, a function is a relation between a given <u>set</u> of elements (the domain) and another set of elements (the codomain), which associates each element in the domain with exactly one element in the codomain. The elements so related can be any kind of thing (words, objects, qualities) but are typically mathematical quantities, such as real numbers. There are many ways to represent or visualize functions: a function may be described by a formula, by a plot or graph, by an algorithm that computes it, by arrows between objects, or by a description of its properties. Sometimes, a function is described through its relationship to other functions (for example, inverse functions). In applied disciplines, functions are frequently specified by tables of values or by formulas. The symbol for the input to a function is often called the independent variable or argument and is often represented by the letter x or, if the input is a particular time, by the letter t. The symbol for the output is called the dependent variable or value and is often represented by the letter y. The function itself is most often called f, and thus the notation $y = f(x)$ indicates that a function named f has an input named x and an output named y. A

function ƒ takes an input, *x*, and returns an output ƒ(*x*). The set of all permitted inputs to a given function is called the domain of the function. The set of all resulting outputs is called the image or range of the function. The range is often a subset of some larger set, called the codomain of a function. Thus, for example, the function $f(x) = x^2$ could take as its domain the set of all real numbers, as its image the set of all non-negative real numbers, and as its codomain the set of all real numbers. In that case, we would describe *f* as a real-valued function of a real variable. It is usual practice in mathematics to introduce functions with temporary names like ƒ. For example, ƒ(*x*) = 2*x*+1, implies ƒ(3) = 7; when a name for the function is not needed, the form *y* = 2*x*+1 may be used. Functions need not act on numbers: the domain and codomain of a function may be arbitrary sets. A function of two or more variables is considered in formal mathematics as having a domain consisting of ordered pairs or tuples of the argument values. For example Sum(*x, y*) = *x* + *y* operating on integers is the function Sum with a domain consisting of pairs of integers. A function is an operation performed on an input (x) to produce an output (y = f(x). The Domain of f is the set of all allowable inputs (x values). The Range of f is the set of all outputs (y values). Formal description of a function typically involves the function's name, its domain, its codomain, and a rule of correspondence. Thus we frequently see a two-part notation, an example being

$$\begin{aligned} f\colon \mathbb{N} &\to \mathbb{R} \\ n &\mapsto \frac{n}{\pi} \end{aligned}$$

Where the first part is read:

- "ƒ is a function from **N** to **R**" (one often writes informally "Let ƒ: *X* → *Y*" to mean "Let ƒ be a function from *X* to *Y*"), or
- "ƒ is a function on **N** into **R**", or
- "ƒ is an **R**-valued function of an **N**-valued variable",

And the second part is read:

- n maps to $\frac{n}{\pi}$

3. A sequence or progression is a list of objects, events or numbers in a definite order of occurrence. Each member of a sequence is called a term. Sequence is a set of object which is listed in a specific order one after another. The elements of sequence are also known as its terms. A sequence of numbers or quantities each term of which different from the succeeding term by a constant amount.
4. A matrix is a rectangular array of elements. it is an ordered collection of numbers arranged in the form of a rectangular array. An ordered collection implies that the element are arranged in such a way that each number will have a fixed position which cannot be altered. According to Sir Arthur Cayley

"A rectangular or a square array of numbers arranged systematically into rows and columns is called a matrix. Structure of the matrix is an arrangement of elements in a particular order which gives it a specific structure

$$A = \begin{array}{c|cccc} & C_{11} & C_{12} & C_{ij} & C_n \\ R_1 & a_{11} & a_{12} & a_{ij} & a_{1n} \\ R_2 & a_{21} & a_{22} & a_{ij} & a_{2n} \\ R_3 & a_{31} & a_{32} & a_{ij} & a_{3n} \\ R_4 & a_{41} & a_{42} & a_{ij} & a_{4n} \end{array}$$

Where C stands for column and R stands for row.

5. Frequency Distribution**:**_A tabular presentation of the data in which the frequencies of values of variable are given along with them is called frequency distribution. It refers to a technique by which similar variables are bunched or grouped in different statistical series. A tabular arrangement of data with corresponding frequency is known as frequency distribution. **"***Frequency distribution is a statistical table which shows the set of all distinct values of the variables arranged in the order of their magnitude, either individually or in groups with their corresponding frequencies side by side*" Thus, a frequency distribution consist of two parts, one part shows magnitude of values whereas other part shows number of times a value or group of value has repeated.

6. Harmonic Mean of a series of positive values is the reciprocal of the arithmetic average of the reciprocals of those values. Harmonic mean is denoted by H or H.M. Computation of harmonic mean in individual series : Let x1, x2,………., xN be the values of variable X, their Harmonic mean is given by –

$$\text{H.M.} = \text{Reciprocal of } \frac{\sum \frac{1}{x}}{N}$$

$$= \frac{N}{\sum \frac{1}{x}}$$

7. If a coin is tossed 10 times we may get 6 heads and 4 tails. the probability of a head is thus 0.6 and that of a tail 0.4 however is that experiment is carry out a large number of times we should expect approximately equal number of heads and tails . An increases i.e. , approaches (infinity), we find that the probability of getting a head or tail approaches 0.5 the probability of an event can thus be defined as the relative frequency with which it occurs in an infinitely large number of trials. If an event occurs a time out of n, its relative frequency is a the value which is approached by a when become infinitely is called the unit of the relative frequency

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Symbolically,

P(A) = limit a/n

Theoretically we can never obtain the probability of an event as given by the above limit. However, in practice we can try to have a close estimate of P(A)based on large number of observations i.e., n

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8. In statistics correlation refers to relationship between any two or more variables like height and weight, rainfall and yield, price and demand, income and expenditure etc. Two variables are said to be correlated if with a change in the value of one variable there arises a change in another variable and vice-versa. **According to A.M. Tuttle,** "Correlation is an

analysis of the covariation between two or more variables." **According to Croxton and Cowden,** "When the relationship is of a quantitative nature, the appropriate statistical tool for discovering and measuring the relationship and expressing it in a brief formula is known as correlation **According to W.I. King,** " Correlation means that between two series or groups of data there exists some casual connection." Thus, from the above definitions, we can say that statistical technique with the help of which we study the extent, nature and significance of association between the two or more variables is known as correlation.

Types of correlation

(A) On the basis of direction:

1. Positive correlation
2. Negative correlation

(B) On the basis of ratio:

1. Linear correlation
2. Non-linear correlation

(C) On the basis of number of variables:

1. Simple Correlation
2. Multiple Correlation
3. Partial Correlation

(D) Perfect correlation:

PROPERTIES OF CORRELATION

1. Correlation requires that both variables be quantitative (numerical).
2. Positive r indicates positive association between the variables, and negative r indicates negative association.
3. The correlation coefficient (r) is always a number between -1 and +1.
4. The correlation coefficient (r) is a pure number without units.
5. The correlation can be misleading in the presence of outliers or nonlinear association.
6. Correlation measures association. But association does not necessarily show causation.

9. Numerical facts which are collected at different points of time take the form of a time series. The unit of time may be a year, a month, a day, an hour etc. Here time is simply a device that enables one to relate all phenomena to a set of common stable reference points. In linear regression, two variables have a cause and effect relationship, one of the variables can be used to estimate the other. But in time series we use time as independent variable to estimate some other dependent variable. A set of numeric observations of the dependent variable, measured at specific points in time in chronological order, usually at equal intervals in order to determine the relationship of time to such variable is known as time series. "A set of data depending on time is called a time series." —***Kenny and Keeping.*** "A time series may be defined as collection of magnitudes belonging to different time periods of some variable or composite of variables such as production of steel, per capita income, gross national income, price of tobacco, index of industrial production." —***Ya-Lun-Chou.*** "A series of values over a period of time is called a time series." ***—P. G. Moore***

10. The method or technique which is used to obtain best solution of decisive problems in production is called linear programming. Here 'best solution' means least cost of production and maximum profit keeping in mind the rules of demand and supply and the alternative uses of the resources if possible) To understand the meaning and

definition of linear programming it is necessary to consider the concept of two words (i) linear and (ii) programming,(i) The word linear' stands for indicating that all relationships involved in a particular problem are of degree one {i.e. linear). For example. y = ax + b is a linear equation or the relation between the variables x and y is linear. In linear programming y = ax + b is also called a homogeneous function of degree one. (ii) The concept, of the word 'programming' is that the optimum solution of the problem is obtained under a system which has a definite plan and is based on mathematical techniques. Thus, we can say. Linear programming is mathematical techniques which are used to determine the optimal allocation of the limited resources, among the competitive activities provided all the relations between all variables are linear represented in the form of linear simultaneous equations or inequalities.

Section-B

Note: Attempt any five questions. This section contains *Eight*long- answer type questions carrying 10 marks each. (5x10=50 Marks).

2. Quantitative Methods are the methods used to quantified analysis of any phenomena. It has three components namely, (i) Elementary concepts in mathematics; (ii) Statistical methods and (iii) Operational Research Methods. Some of the important methods of each of its components are as under:

Elementary concepts in mathematics

(i) Set Theory;
(ii) Permutations and Combinations;
(iii) Binomial Theorem;
(iv) Equations;
(v) Functions and Variables;
(vi) Elements of Calculus;
(vii) Classical Optimisation Techniques;
(viii) Vectors, Matrices and Determinants;
(ix) Mathematics of Finance etc

Statistical methods

(i) Presentation and Analysis of Statistical Data;
(ii) Measures of Central Tendency and Location;
(iii) Measures of Dispersion, Skewness and Kurtosis;
(iv) Probability;
(v) Random Variables and Probability Distributions;
(vi) Sampling and Sampling Distributions;
(vii) Testing of Hypothesis;
(viii) Decision Making under Uncertainty;
(ix) Correlation, Regression and multivariate Analysis;
(x) Time Series Analysis;
(xi) Index Numbers etc

Operational Research Methods

(i) Linear Programming;
(ii) Transportation problems;

(iii) Theory of Games;
(iv) Network Analysis;
(v) Waiting Lines;
(vi) Inventory Controls;
(vii) Replacement Models;
(viii) Sequencing Models;
(ix) Integer, Dynamic and Goal programming;
(x) Simulation of Management systems etc

(Adequate description to the above should be given)

3. ARITHMETIC PROGRESSION An Arithmetic progression is a sequence of number in which each term is obtained by adding a fixed number to the preceding term except the 1st term. This fixed no. is called the common difference {d}, this number can be positive, negative or zero. The general form of an Arithmetic Progression is

a , a +d , a + 2d , a + 3d ……………., a + (n-1)d

Where 'a' is first term and 'd' is called common difference

PROPERTIES OF ARITHMETIC PROGRESSION

a) If a constant is added to or subtracted from each term of an A.P., then the resulting sequence is also an A.P. with the same common difference

b) If each term of a given A.P. is multiplied or divided by a non-zero constant k, then the resulting sequence is also air A.P. with common difference kd or d/k, where d is the common difference of the given A.P.

c) Three numbers a, b, c are in A.P. if 2b= a + c

d) If the terms of an A.P. are chosen at regular intervals, then they form an A.P.

FORMULA FOR FINDING THE Nth term

a = an+(n-1)d

Where,

- a is the 1st term
- d is the common difference
- an ,is a number which comes at the nth term.
- n is the number of term

Sum of n terms = n/2(a+l) = n/2[2a +(n-1)d]

Let 124 be the nth term of an A.P., then a_n = 124

so, 124 = a +(n-1)d

124 = 4+(n-1)5

124 = 4+5n-5

124 = 5n-1

125 = 5n

n = 25

4. A = $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

we have the matrix of cofactors

$$[A_{ij}] = \begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & -6 \end{bmatrix}$$

since |A| = -5 we have

A^{-1} **= 1/-5 [Aji]**

$$= 1/5 \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0.8 & 1.8 \\ 3 & -0.8 & -2.8 \\ 2 & 0 & 1 \end{bmatrix}$$

5. SOLUTION

Class (X)	MID VALUE x	Frequency f	fx	U= (x-15)/10	fu
0-10	5	12	60	-1	-12
10-20	15	15	225	0	0
20-30	25	28	700	1	28
30-40	35	25	875	2	50
40-50	45	20	900	3	60
TOTAL		∑f= 100	∑fx = 2760		∑fu = -126

DIRECT METHOD

$(X) = \frac{\sum fx}{\sum f} = \frac{2760}{100}$

= 27.6

SHORTCUT METHOD

$(X) = a + \frac{\sum fu}{\sum f} \times i$

$= 15 + \frac{126}{100} \times 10 = 15+12.6 = 27.6$

6. M

X	dx = (X-5)	dx^2	Y	dy = (Y – 12)	dy^2	dxdy
1	-4	16	9	-3	9	12
2	-3	9	8	-4	16	12
3	-2	4	10	-2	4	4
4	-1	1	12	0	0	0
5	0	0	11	-1	1	0
6	1	1	13	1	1	1
7	2	4	14	2	4	4
8	3	9	16	4	16	12
9	4	16	15	3	9	12
	Σdx = 0	$\Sigma dx^2 = 60$		Σdy = 0	$\Sigma dy^2 = 60$	Σdxdy=57

$\bar{X}$ = Σx/n = 45/9 = 5; **$\bar{Y}$ =** Σy/n = 108/9 = 12 **dx = (X-5), dy= (Y-12)**

4. Finally apply the following formula to calculate correlation :

$$r = \frac{\Sigma dxdy - \frac{\Sigma dx \times \Sigma dy}{N}}{\sqrt{\Sigma dx^2 - \frac{(\Sigma dx)^2}{N}} \times \sqrt{\Sigma dy^2 - \frac{(\Sigma dy)^2}{N}}}$$

r = 57/610 = 095

7.

X	Y	x = X – 165	y = Y – 165	x^2	Y^2	xy
170	168	5	3	25	9	15
167	167	2	2	4	4	4
162	166	-3	1	9	1	-3
163	166	-2	1	4	1	-2
167	168	2	3	4	9	6
166	165	1	0	1	0	0
169	168	4	3	16	9	12
171	170	6	5	36	25	30
164	165	-1	0	1	0	0
165	168	0	3	0	9	0
Total		14	21	100	67	62

Let A = 165 and B = 165 be the assumed means of X and Y series respectively then, x = X-165 and y = Y – 165

$\bar{X}$ = 165 + 14/10 = 166.4 and $\bar{Y}$ = 165 + 21/10 = 167.1,

The regression equation of Y on X is given by Y- $\bar{Y}$ = byx (X-$\bar{x}$),

Where byx = (n $\Sigma xy - \Sigma x\ \Sigma y)/n\ \Sigma x^2 - (\Sigma x)^2$ = (10 X 62 – 14 X 21)/10 X 100 - 14^2 **= 326/804 = 0.405**

The required regression equation is

Y – 167.1 = 0.405(X-166.4)

Or, Y = 0.405X + 99.708

8. **Let the equation of the straight line of best fit with the origin at the middle year 2010 and unit of X as 1 year be**

Y = a + bX

By the method of least squares, the value of a and b are given by,

a = ΣY/N and b = ΣXY/ΣX^2 **and N = 5**

Year	Sales ('000 Rs.)=Y	X	X^2	XY
2008	35	-2	4	-70
2009	56	-1	1	-56
2010	79	0	0	0
2011	80	1	1	80
2012	40	2	4	80
Total	ΣY = 290	ΣX=0	Σ X^2=10	Σ XY= 34

a = ΣY/N = 290/5 = 58 and b = ΣXY/ΣX^2 **= 34/10 = 3.4**

the required equation of the straight line of best fit with the origin at the middle year 2010

Y = 58 + 3.4X with 2010 as the base year.

The trend values will be

Year	X	Trend Vales 58 + 3.4X
2008	-2	51.2
2009	-1	54.6
2010	0	58.0
2011	1	61.4
2012	2	64.8

9. **Solution of LPP by Graphical Method :** The determination of the solution space shows how a two variable LP model is solved graphically. Although two-variable models rarely occur in practice (where a typical LP model may include thousands of variables and constraints), the ideas gleaned from the graphical procedure lay the foundation for the development of the general solution technique (called the simplex method).

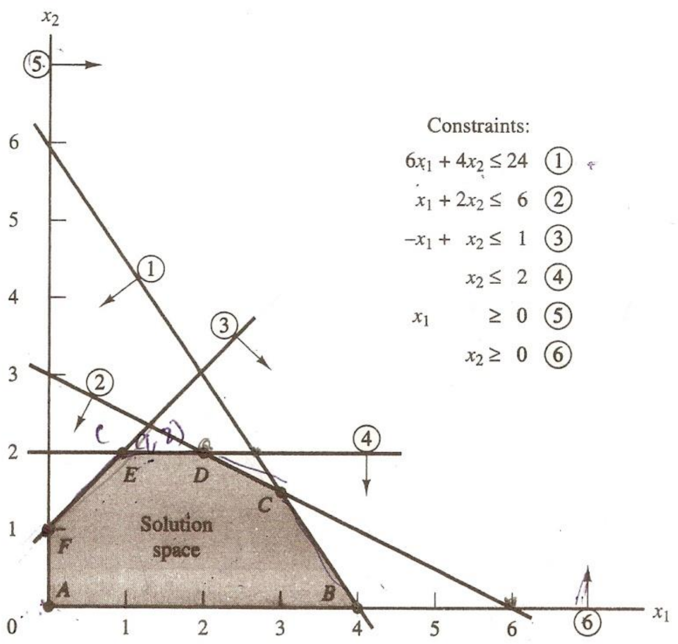
1. The determination of the solution space that defines the feasible solutions that satisfy all the constraints of the model.
2. The determination of the optimum solution from among all the points in the feasible solution space.

The procedure is described with the help of an example.

We use the Reddy Mikks model to illustrate the two steps of the graphical procedure.

Step 1. Determination of the Feasible Solution Space :

First, as shown in Figure let the horizontal axis x1 and the vertical axis x2 represent the exterior paint and interior paint variables, respectively. Next, consider the no negativity restrictions $x_1 \geq 0$ and $x_2 \geq 0$. these two constraints restrict the solution space area to the first quadrant (which lies above the x_1 – axis and to the right of the x_2- axis).



The easiest way of accounting for the remaining four constraints is to replace the inequalities with equations and then plot the resulting straight lines. For example, the inequality $6x_1 + 4x_2 \le 24$ is replaced with the straight line $6x_1 + 4x_2 = 24$. To plot this line, we need two distinct points, which can be secured by first setting $x_1 = 0$ to obtain, $x_2 = 24/4 = 6$, and then setting $x_2 = 0$ to obtain $x_1 = 24/6 = 4$. Thus, the line passes through the two points (0,6) and (4,0) as shown by line (1) in Figure.

In spite of having many advantages and wide areas of applications, there are some limitations associated with this technique. These are given below.

1. Relationships among two variables as linear can only be solved by this method. However, generally, neither the objective functions nor the constraints in real-life situations concerning business and industrial problems are linearly related to the variables. And with two variables.
2. While solving an LP model with this method, there is no guarantee that we will get integer valued solutions.
3. It does not take into consideration the effect of time and uncertainty.
4. Parameters appearing in the model are assumed to the constant but in real life situations, they are frequently neither known for constant.
5. It deals with only single objective, whereas in real-life situations we may come across conflicting multi-objective problems.